Program for the course "Dynamical Systems"

(Lecturer Boris Kruglikov)

1. Introduction to the course. Actions of groups G. Examples of discrete dynamics. Actions of the group of reals $G = \mathbb{R}$. Vector fields v. Cauchy theorem on existence and uniqueness in ODE $\dot{x} = v(x)$, $x \in U \subset \mathbb{R}^n$. Flows $\varphi(t, x)$, critical points and phase portraits on the example of ideal pendulum equation and its approximations. Integrals F of ODE.

2. Liapunov and asymptotically stable solutions. Study of the stability of any solution is equivalent to the study of stability for a critical point of ODE. Structure of solutions of linear ODE $\dot{x} = Ax$ and criterium of stability. Liapunov theorem on the stability of nonlinear equations by the first approximation $\dot{x} = Ax + \alpha(x)$, $\|\alpha\| = o(\|x\|)$, $x \in \mathbb{R}^n$.

3. Floquet theory of linear ODE with periodic coefficents $\dot{x} = A(t)x$, $x \in S^1$. Stability in terms of the Floquet multipliers μ_i. Problem of linearization.

4. Connection between discrete and continuous dynamics: time t-maps, first return maps, suspensions.

5. Some general properties of dynamical systems: Periodicity of trajectories, Dense orbits, Minimal dynamical systems, Topological mixing, Structural stability.

6. Continuous time dynamical systems: Linear flows on tori T^n, Gradient flows $v = \operatorname{grad}_S F$ on hypersurfaces in Euclidean space $S \subset \mathbb{R}^{n+1}$, Hamiltonian systems (skew-gradient flows) in $\mathbb{R}^{2n}$. Action of $\mathbb{R}^n$ on the phase space of integrable Hamiltonian systems. Liouville theorem on integrable Hamiltonian systems $v = \operatorname{sgrad}_\Omega F$. Stability of the minimax critical points and minimax periodic trajectories.

7. Discrete time dynamical systems: Rotations of the circle S^1, Shifts of the tori T^n, Expanding mappings $E_m : S^1 \to S^1$, Hyperbolic maps of tori $T^2 \to T^2$, Symbolic dynamics on Ω_N, Topological Markov chains σ_A, Smale horseshoe.

8. Invariants: Exponential growth of the periodic points number $p(f)$, zeta-function $\varsigma_f(z)$ and topological entropy $h_{\text{top}}(f)$.

9. Calculations of the invariants of dynamical systems for our examples.

10. The concept of conjugation. Conjugation of discrete and smooth dynamical systems. Topological and differentiable conjugations.

11. Conjugation of the diffeomorphisms of the circle S^1. Rotation number μ. Denjoy theorem. Consequence for the flows on tori.

12. Conjugation and the semi-local analysis: Coding, Cantor sets and expanding mappings of the circle of degree k.

13. Conjugation of the critical points to their linearizations: Theorems of Poincare (formal and analytic), Grobman-Hartman (C^0) and Sternberg (C^k, C^∞). Invariant manifolds for maps and flows. Stable and unstable manifolds W^s, W^u of the critical point. Stable manifold theorem.

14. ω- and α-limit sets. Foliation of these sets by the solutions. Particular cases: Critical points and Periodic trajectories. Limit cycles. Poincare-Benedixon theorem.

15. Two-dimensional dynamics (M^2, v). Homoclinical points. Bifurcations of the phase portraits. Problem of conjugation.

16. Conclusion: Some words about chaos, perturbation theory, asymptotic decomposition and holomorphic dynamics.

* This program is intended but is not final, some changes can appear during the course.