Program for the course "Dynamical Systems" *(2002)*

(Lecturer Boris Kruglikov)

1. Basics of differential equations: Vector fields v. Cauchy theorem on existence and uniqueness of ODE $\dot{x} = v(x)$, $x \in U \subset \mathbb{R}^n$. Flows $\varphi(t, x)$, critical points and phase portraits. Numerical methods. Examples (ideal pendulum equation and its approximations).

2. Non-linear oscillations: Perturbation theory and averaging methods. Example 1: driven and damped Duffing model. Example 2: parametric resonance.

3. General concept of a dynamical system: Prerequisitives from topology. Lie group action. Discrete and continuous time dynamical systems. Connections between different type dynamical systems: Time-T maps, Poincaré map, Suspension.

4. Structure of solutions of linear ODE $\dot{x} = Ax$ and criterium of stability. Lyapunov and asymptotic stability for nonlinear by the first approximation equations $\dot{x} = Ax + \alpha(x)$, $\|\alpha\| = o(\|x\|)$, $x \in \mathbb{R}^n$.

5. Floquet theory of linear ODE with periodic coefficients $\dot{x} = A_t x$, $A_{t+1} = A_t$, $x \in S^1$. Stability in terms of the Floquet multipliers μ_i. Problem of linearization.

6. General stability theory: Abstract theory of Lyapunov exponents in $\mathbb{R}^n$. Lyapunov regularity. Forward, backward and Lyapunov regularity. Lyapunov criterion of regularity. Lyapunov and Malkin stability theorems.

7. Lyapunov exponents for non-autonomous systems $\dot{x} = A_t x$ in $\mathbb{R}^n$ and for general dynamical systems. Elements of Ergodic theory. Multiplicative ergodic theorem. Hyperbolicity from measure point of view.

8. Hyperbolicity from topological point of view. Grobman-Hartman theorem. λ-lemma. Stable and unstable manifolds.

9. Normal forms of dynamical systems. Resonances. Poincare, Poincare-Siegel, Sternberg and Poincare-Dulac theorems.

10. Topological characterizations of dynamical systems: minimality, topological transitivity, sensitivity to initial conditions, density of periodic points. Chaotical dynamical systems.

11. One dimensional dynamical systems: Rotation of the circle S^1. Expanding maps E_m. Maps of the interval $f : I \to I$. Quadratic maps F_λ. Cantor set. Symbolic dynamic. Topological Markov chains $(\Omega_A^{(+)}, \sigma_A^{(+)})$.

12. Higher-dimensional dynamical systems: Shifts on the tori $(\mathbb{T}^n, R_\gamma)$. Hyperbolic automorphisms of tori $(\mathbb{T}^n, A)$. Smale horseshoe. Plykin attractor. Coding.

13. Invariants from the periodic points: Exponential growth number $p(f)$ and zeta-function $\varsigma_f(z)$. Calculations.

14. Topological entropy $h_{top}(f)$, $h_{top}(\varphi_t)$. Properties and calculations. Elements of measure entropy h_μ. Relation to the Lyapunov characteristic exponents χ_i (Ruelle inequality, Pesin formula). Variational principle.

15. Conjugacy and semi-conjugacy. Structural stability. Proof of structural stability for some dynamical systems. Are structurally stable systems typical?

16. Homeomorphisms of the circle. Rotation number. Denjoy's theorem. Maps of higher degree. Conjugacy problem.

17. Qualitative theory of continuous time dynamical systems in dimension 2. ω- and α-limit sets. Foliation of these sets by the solutions. Particular cases: Critical points and Periodic trajectories. Limit cycles. Poincare-Benedixon theorem. Morse-Smale flows.

18. Homoclinic and heteroclinic points/trajectories. Transversal homoclinic points imply chaotisity.

References

[1] A. Katok, B. Hasselblatt, "Introduction to the modern theory of dynamical systems", Cambridge University Press, 1995.

[2] R.L. Devaney, "An introduction to chaotic dynamical systems". Second edition. Addison-Wesley Publishing Company, 1989.

[3] L. Barreira, Ya. Pesin, "Lyapunov exponents and smooth ergodic theory", University Lecture Series, 23, A.M.S., Providence, RI, 2002

[4] L. Cesari, "Asymptotic behavior and stability problems in ordinary differential equations". Third edition. Springer-Verlag, 1971.