#### **Dmitry Alekseevsky**

## "From cubic polynomials to quaternionic Kähler manifolds"

## (based on joint papers with V. Cortes and Th. Mohaupt)

I will give a purely geometric description of a map which associates with a "very special real manifold", which is the Riemannian manifold M(f) defined by a Hessian metric Hess(f) on a level set of a cubic polynomial f, a special Kähler manifold and a map, which associates with a special Kähler manifold a quaternionic Kähler manifold.

These maps were defined by physisits De Wit and Van Proyeyn in context of sypergravity and called in physical literature r map and c map respectively. They correspond to dimensional reduction from d=5 supergravity to d=4 supergravity and then to d=3 supergravity.

Some years ago, we get a simple geometric description of r map, and recently we have obtained (not so simple) geometric description of c map.

## **Glenn Barnich**

# "Classical and gravitational aspects of the AdS3/CFT2 correspondence"

It is shown in detail how conformal symmetries emerge in 3 dimensional asymptotically anti-de Sitter spacetimes. The crucial role of the Lie algebroid associated to gauge systems is pointed out. Centrally extended representations of the asymptotic symmetries on solution space and in terms of canonical generators are also discussed.

## Pavel Bibikov

## "Differential invariants and classification of linear actions on homogeneous forms"

(joint work with Valentin Lychagin)

no abstract is given

## Evgeny Ferapontov (Loughborough University, UK)

## "On the central quadric ansatz: integrable models and Painleve reductions"

## (based on joint paper with B. Huard and A. Zhang)

It was observed by Tod and later by Dunajski and Tod that the Boyer-Finley (BF) and the dispersionless Kadomtsev-Petviashvili (dKP) equations possess solutions whose level surfaces are central quadrics in the space of independent variables (the so-called central quadric ansatz). It was demonstrated that generic solutions of this type are described by Painleve equations P3 and P2, respectively.

The aim of this talk is threefold:

- Based on the method of hydrodynamic reductions, we describe integrable models possessing the central quadric ansatz. This leads to the five canonical forms (including BF and dKP).

- Applying the central quadric ansatz to each of the five canonical forms, we obtain all Painleve equations P1-P6, with P6 corresponding to the generic case of our classification. - We argue that solutions coming from the central quadric ansatz constitute a subclass of two-phase solutions provided by the method of hydrodynamic reductions.

# Adam Hlavac

## "Some results concerning the constant astigmatism equation"

## (joint work with Michal Marvan)

We continue investigation of the constant astigmatism equation  $z_{yy} + (1/z)_{xx} + 2 = 0$ . We newly interpret its solutions as describing spherical orthogonal equiareal patterns, with relevance to twodimensional plasticity. We show how the classical Bianchi superposition principle for the sine-Gordon equation can be extended to generate an arbitrary number of solutions of the constant astigmatism equation by algebraic manipulations. As a by-product, we show that sine-Gordon solutions give slip line

## Hilja Lisa Huru

# "Non-commutative Galois extensions"

# (joint work with Valentin Lychagin)

Galois extensions of a field F can be quantized as group modules of their Galois group G. We will show that there is a 1-1 correspondence between the set of quantizers of the and the second cohomology group of the dual of G with coefficients in the group of units of F. The quantizations and resulting non-commutative extension will be presented for examples of cyclic extensions.

## Alexei Kushner

## "Feedback Classification of Hamiltonian Systems"

## (joint work with Valentin Lychagin)

We consider the action of symplectic feedback transformations on control Hamiltonian systems. We study differential invariants of the pseudogroup of feedback symplectic transformations, which we call Petrov invariants, and show that the algebra of invariants carries a natural Poisson structure and a central derivations. This structure allows us to classify regular control Hamiltonian systems.

## Andrei Marshakov

## "Cluster varieties and integrable systems"

## (based on joint unpublished work with V.Fock)

I describe a class of integrable systems on Poisson submanifolds of the affine Poisson-Lie groups. This class of integrable systems coincides with the constructed by Goncharov and Kenyon out of dimer models on a twodimensional torus and is classified by Newton polygons. The correspondence between the Weyl group elements and polygons demonstrates, that every integrable model admits many realisations on the Poisson-Lie groups.

## Oleg Morozov

## "SDiff(2) and uniqueness of the Plebański equation"

(joint work with Boris Kruglikov)

The group of area preserving diffeomorphisms showed importance in the problems of self-dual gravity and integrability theory. We discuss how representations of this infinite-dimensional Lie group can arise in mathematical physics from pure local considerations.

Then using Lie algebra extensions and cohomology we derive the second Plebański equation and its geometry. We do not use Kähler or other additional structures but obtain the equation solely from the geometry of area preserving transformations group. We observe that on this way there arise 4 copies of SDiff(2). A similar story happens with other integrable Monge-Ampère equations in 4D.

# Valentine Ovsienko

## "Linear differential operators on contact manifolds"

# (joint work with Charles Conley)

We consider differential operators between sections of arbitrary powers of the determinant line bundle over a contact manifold. We extend the standard notions of the Heisenberg calculus: noncommutative symbolic calculus, the principal symbol, and the contact order to such differential operators. Our first main result is an intrinsically defined "subsymbol" of a differential operator, which is a differential invariant of degree one lower than that of the principal symbol. In particular, this subsymbol associates a contact vector field to an arbitrary second order linear differential operator. Our second main result is the construction of a filtration that strengthens the well-known contact order filtration of the Heisenberg calculus.

# Peter Olver

# "Lie pseudo-groups and moving frames"

In this talk, I will present a new approach to study Lie pseudo-groups, based on the variational bicomplex and a new generalization of the Cartan theory of moving frames. Applications will include new constructive algorithms for the Maurer-Cartan forms and the structure equations, as well as the differential invariants and invariant differential operators and forms. The consequential recurrence formulae reveal the complete structure of the differential invariant algebra of the pseudo-group. Applications to differential equations and variational problems arising in geometry and physics will be discussed.

## **Maxim Pavlov**

# "Reductions of kinetic equations to finite component systems"

We consider two approaches for extraction of finite component systems from kinetic equations. The first method is based on the theory of generalized functions, which in simplest case is nothing but the so called multi flow hydrodynamics well known in plasma physics. An alternative is the so called moment decomposition method successfully utilized for hydrodynamic chains. We prove that both approaches lead to the same finite component systems.

The method of hydrodynamic reductions, successfully utilized in the theory of integrable hydrodynamic chains, is applied to the local and nonlocal kinetic equations. N component reductions parameterized by N-1 arbitrary constants for non-hydrodynamic chain arising in the theory of high frequency nonlinear waves in electron plasma are found. These evolution dispersive systems equipped by a local Hamiltonian structure possess periodic solutions.

# Prikhod'ko

"Littlewood polynomials and soliton-like behaviour of a free quantum particle constrained on the circle"

We study dynamical properties of a class of quantum systems having multiplicative self-similarity property of the spectrum. This class is known to be modelled by certain non-linear Schrödinger equations and, in algebraic terms, by q-deformed Heisenberg-Weyl algebras. Our main purpose is to investigate the connection between the behaviour of quantum systems and analytic properties of Littlewood polynomials, well-known in calculus and number theory. It is shown that for any compact set in the real line there exists an exponential sum which is flat on this set according to integral norm. This effect is established via the study of soliton solutions of quantum systems with multiplicative symmetry of the discrete spectrum. We also discuss combinatorial complexity phenomena as well as topological properties of the induced dynamical systems.

## Pedro de M Rios

# (joint work with Eldar Straume)

# "On symbol correspondences for spin systems"

We investigate general symbol correspondences for spin systems, that is, SU(2)-equivariant linear maps from (n+1)X(n+1) complex matrices to complex functions on the 2-sphere that preserve reality and normalization.

We classify all such correspondences, also exhibiting concrete constructions, and study the products of functions on the 2-sphere which are induced from the operator product. In particular, we present various expressions for such products and study some of their asymptotic properties, for large n.

## Stefan Rosemann

# "Kähler manifolds with high degree of mobility"

Two Kähler metrics are called h-projectively equivalent if their h-planar curves coincide. These curves can be seen as a natural generalisation of geodesics on complex manifolds. For a fixed Kähler metric, the dimension of the space of h-projectively equivalent metrics is called the degree of mobility.

In my talk, I show that for a Kähler metric of degree of mobility at least three, the h-projectively equivalent metrics can be equivalently described as parallel, hermitian (0,2)-tensors on a C\*-bundle over the manifold. In particular, this will lead to a simple formula for the possible values of the degree of mobility of a Kähler metric.

## Artur Sergyeyev

## "An explicit construction of recursion operators for a class of multidimensional dispersionless PDEs"

We present a new explicit construction of recursion operators for integrable multidimensional dispersionless systems which admit a Lax representation in terms of vector fields of a certain special form.

The examples include the Manakov-Santini system in N+4 independent and N dependent variables, where N is an arbitrary natural number, the dispersionless Hirota equation, and many more.

# **Georgy Sharygin**

# "Full symmetric Toda flow and the Bruhat order on permutation groups"

## (based on joint work with Yu.Chernjakov and A.Sorin)

We investigate the well-known integrable system on classical groups, full symmetric Toda flow, which generalizes the well-known three-diagonal matrix case. We describe the set of singular points of this system and describe the phase transformations, i.e. the trajectories, connecting these points. It turns out that these points can be identified with the Weil group of SL(n) and (in low-dimensional cases, where we can compute

everything "by hands"), that they are connected by trajectories iff the corresponding elements are Bruhatcomparable. I will explain this result in low dimensions and give few hints of why this should be true in a general case.

# Vadim Shurygin

# "Action of contact transformation group on the 2nd order ODEs which are polynomial in 2nd derivative"

S.Lie showed that any two 2nd order ODEs y''=f(x,y,y') are contact equivalent. The problem of equivalence of such ODEs under the action of the pseudogroup of point transformations was studied in several papers. Lie proved they are linearizable if only if they are cubic in the first derivative.

A.Tresse obtained the complete set of differential invariants for general 2nd order ODEs. In the paper of B.Kruglikov the results of Tresse were reviewed using the modern geometric approach. Moreover, Kruglikov described the algebra of absolute differential invariants and proved the equivalence theorem.

In the present talk we study the problem of local contact equivalence of the 2nd order ODEs that are polynomial in the second derivative. For the quadratic polynomials we show how this problem can be reduced to the point equivalence of some ODEs of the form y''=f(x,y,y') associated with the given ODE.

For the cubic polynomials we consider the situation when the equation y''=f(x,y,y'), associated to the first two roots of this polynomial, is linearizable. The group of point transformations, which preserves such an equation, is the 8-dimensional Lie group SL(3).

Thus, we reduce our problem to the problem of SL(3)-equivalence of the 2-parametric families of curves, which are integral curves of Cartan distribution. We give the complete description of the algebra of differential invariants and prove the equivalence theorem. We generalize this to the problem of SL(3)-equivalence of arbitrary 2-parametric families of curves in 3-dimensional space.

## **Thomas Strobl**

## "Lie algebroid symmetries and their gauging"

## (joint work with V. Salnikov)

Dirac sigma models are a generalization of Poisson sigma models, two-dimensional topological field theories that can be associated to a Dirac manifold given some auxiliary data. A particular example of them is the gauged WZW-model, which results from the usual WZW model by gauging a finite-dimensional symmetry group. Adapting a generalization of equivariant cohomology to the context of Lie algebroid symmetries developed together with A. Kotov, we show that the part of the Dirac sigma model that does not depend on the auxiliary data, can be obtained from gauging this infinite dimensional symmetry group. In contrast to the standard procedure of equivariant cohomology, one obtains a finite number of gauge fields only.

## **Dennis The**

## "The gap phenomenon in parabolic geometries"

## (joint work with Boris Kruglikov)

Many geometric structures (such as Riemannian, conformal, CR, projective, systems of ODE, and various types of generic distributions) admit an equivalent description as Cartan geometries. For Cartan geometries of a given type, the maximal amount of symmetry is realized by the flat model. However, if the geometry is not (locally) flat, how much symmetry can it have?

Understanding this "gap" between maximal and submaximal symmetry in the context of parabolic geometries is the subject of this talk. We show how representation-theoretic considerations involving Kostant's version of the Bott-Borel-Weil theorem and Tanaka prolongation lead to restrictions on the submaximal dimension. In particular, I'll discuss conformal geometry as well as the (G2) geometry of generic rank two distributions in dimension five.

#### **Alexander Vasiliev**

## "Euler-Arnold equations in sub-Riemannian geometry on Teichmueller space and curve"

We consider the group of orientation-preserving diffeomorphisms of the unit circle and its central extension, the Virasoro-Bott group, with their respective horizontal distributions chosen to be Ehresmann connections with respect to a projection to the smooth universal Teichmueller space and the universal Teichmueller curve associated to the space of normalized univalent functions. We find formulas for the normal geodesics with respect to the pullback of the invariant Kählerian metrics, namely, the Velling-Kirillov metric on the class of normalized univalent functions and the Weil-Petersson metric on the universal Teichmueller space. The geodesic equations are sub-Riemannian analogues of the Euler-Arnold equation and they lead to the CLM, KdV and other known non-linear PDEs.

## **Peter Vassiliou**

## "Wave Maps and Darboux Integrability"

The study of wave maps from the point of view of their complete integrability is well known. Here we initiate the discussion of the Darboux integrability of wave map systems. We point out the existence of a new integrable metric and construct explicit representation formulas. Finally we study the initial value problem for wave maps into a non-constant curvature metric whose governing equation is Darboux integrable.

## Travis Willse

## "Nurowski's conformal structures and highly symmetric 2-plane fields on 5-manifolds"

Nurowski showed that any generic 2-plane field (equivalently, a generic rank 3 Pfaffian system) on a 5manifold induces a canonical signature (2,3) conformal structure on its underlying manifold. After reviewing Nurowski's construction, we show that the conformal structures induced by a class of highly symmetric 2plane fields, including the submaximally symmetric 2-plane fields (which can be realized as differential equations  $z'=(y'')^m$  and whose infinitesimal symmetry algebra has rank 7), admit some interesting additional structure. As an application, we show that these conformal structures induce normal conformal connections and metrics with unusual holonomy groups.

## Valeriy Yumaguzhin

## "The Cauchy problem for the Einstein equation. Cohomological uniqueness of formal solutions"

## (joint work with Valentin Lychagin)

The Cauchy problem for the Einstein equation is investigated for the case when normals to the initial hypersurface have non-zero lengths. It is known that this problem is not correct. We propose a differential invariant of initial data on the initial hypersurface that is an obstruction to exist formal solutions of the problem. When this invariant is trivial, we construct a cohomologously unique formal solution of the problem by the corresponding spectral sequence.