Half-flat causal structures in dimension four and integrable systems

Omid Makhmali

IMPAN, Warsaw

A causal structure is given by a field of tangentially nondegenerate projective hypersurfaces over a manifold, which is an extension of conformal pseudo-Riemannian structures [Mak18]. Using Cartan's method of equivalence, we will solve the local equivalence problem for causal structures and give a geometric interpretation of their fundamental invariants. Our main point of focus in this talk will be four dimensional causal geometries that are half-flat and locally isotrivial.

Half-flat causal structures are characterized by the existence of a 3-parameter family of *null surfaces*. They turn out to be equivalent to 3-dimensional path geometries via an extension of the standard twistor correspondence. We extend conformal notions such as *principal null planes* and *ultra-half-flatness*, as defined in [Cal14, DW07], to the causal setting. After showing that the unique submaximal model that does not descend to a conformal structure is *Cayley-isotrivially flat*, we will focus on *Cayley structures*. We explore several geometries arising from this class of causal geometries. Finally, following [DFK15], we formulate such structures in terms of a dispersionless Lax pair and study the resulting system of PDEs.

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